F&FM BPES

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**Homework 1 - Solutions**

**Part I: Multiple Choice**

1. The bid price of a Treasury bill is \_\_\_\_\_\_\_\_\_\_.

A) the price at which the dealer in treasury bills is willing to sell the bill

B) the price at which the dealer in treasury bills is willing to buy the bill

C) greater than the ask price of the treasury bill expressed in dollar terms

D) the price at which the investor can buy the treasury bill

Answer: B. It is the price at which the dealer is willing to buy the bill. Likewise, the ask price would be the price at which the dealer is willing to sell the asset.

2. A 10-year Treasury bond with an 8% coupon rate should sell for \_\_\_\_ a 10 year Treasury bond with a 12% coupon rate.

A) less than

B) more than

C) the same as

D) indeterminate

Answer: A. Given that the price of the bond depends on the value of coupons that the bond generates; the larger is the value of the coupons the higher should be the price someone would have to pay for that bond. Note that here we assume that everything else, in particular yield, is the same for the two bonds.

3. You purchased 100 shares of ABC common stock on margin at $50 per share. Assume the initial margin is 50% and the maintenance margin is 30%. Below the stock price of \_\_\_\_\_\_\_\_\_\_ you would get a margin call. Assume the stock pays no dividend and ignore interest on margin.

A) $35.71

B) $42.86

C) $53.57

D) $57.14

Answer: A. To understand this, note that the investor gets a margin call whenever the margin level in her account drops below the maintenance margin of 30%. Thus, we need to find the price at which the margin is equal to 30%. Any price below that would trigger the call. Margin = Equity/Assets. The value of assets equals 100\*P, where P is the price we would like to find. Since the initial margin is 50% and the total value of 5000 worth of assets has been purchased, this implies that someone has used 2500 of loan to support the margin. Hence, the equity value of the account would be equal to 100\*P-2500 at the critical price of P. Note that loan stays constant in such an account. We thus need to solve the equation: (100\*P-2500)/100\*P=0.3. This gives us the answer of P=35.71.

4. Assume you purchased 200 shares of XYZ common stock on margin at $80 per share from your broker. If the initial margin is 60%, the amount you borrowed from the broker is \_\_\_\_\_\_\_\_\_\_.

A) $4000

B) $6400

C) $9600

# D) $16000

Answer: B. If the investor maximizes his debt capacity he would use exactly as much debt as to make his margin level to be equal to 60%. This means that Equity/Value=0.6. Given that the value of the asset is 200\*80=16000, this implies that the equity portion is 9,600 and as such the loan portion is 16,000-9,600=6,400.

5. Consider a Treasury bill with a rate of return of 5% and the following risky securities:

Security A: E(r) = .15; 2 = .0400

Security B: E(r) = .10; 2 = .0225

Security C: E(r) = .12; 2 = .1000

Security D: E(r) = .13; 2 = .0625

The investor must develop a complete portfolio by combining the risk-free asset with one of the securities mentioned above. The security the investor would choose as part of his complete portfolio would be \_\_\_\_\_\_\_\_\_\_.

A) security A

B) security B

C) security C

D) security D

Answer: A. Note that we are always mixing one risky asset with a risk-free asset, so the CAL is steepest when the risky security has the highest Sharpe ratio. SR=(E(r)-rf)/ Among the four options, A has the highest Sharpe ratio.

6. An investor invests 40% of her wealth in a risky asset with an expected rate of return of 15% and a variance of 4% and 60% in a Treasury bill that pays 6%. Her portfolio's expected rate of return and standard deviation are \_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_ respectively.

A) 8.0%, 12%

B) 9.6%, 8%

C) 9.6%, 10%

D) 11.4%, 12%

Answer: B. We can start by noticing that the weight the investor allocates in a risky asset is w=0.4, and the weight in a risk-free asset is 1-w=0.6. The expected return is a weighted average of the individual expected returns. Hence, E(r)= 0.4\*15%+0.6\*6%=9.6%. The risk of such portfolio, expressed by its standard deviation, is 

Use the following to answer questions 7-9:

You invest $100 in a complete portfolio. The complete portfolio is composed of a risky asset with an expected rate of return of 12% and a standard deviation of 10% and a Treasury bill with a rate of return of 5%.

7. \_\_\_\_\_\_\_\_\_\_ of your complete portfolio should be invested in the risk-free asset if you want your complete portfolio to have a standard deviation of 9%.

A) 100%

B) 90%

C) 50%

D) 10%

Answer: D. A standard deviation of any portfolio mixing a risky asset with a risk-free asset is equal to Pwrw is a weight allocated to a risky portfolio and ris a standard deviation of a risky asset. If we assume that the risk of a portfolio equals 9% it means that 0.09=w\*0.1. Hence, w=90%. Since we are interested in a weight allocated to the risk-free asset, this means 1-w=10%.

8. A portfolio that has an expected outcome of $115 could be formed if you \_\_\_\_\_\_\_\_\_\_.

A) invest $100 in the risky asset

B) invest $80 in the risky asset and $20 in the risk-free asset

C) borrow $42.86 at the risk-free rate and invest $142.86 in the risky asset

D) borrow $33.33 at the risk-free rate and invest $133.33 in the risky asset

Answer: C. Note that the return on a portfolio with a terminal cash flow of 115 is equal to r=115/100-1=15%. So, now we have to use a formula for the expected return of a portfolio. It is E(r)=w\*E(rr)+(1-w)\*rf . Plugging in the numbers, we have 15%=w\*12%+(1-w)\*5%. The solution is w=1.4286. This means we are buying on margin with a total investment of 142.86. Since we had only 100 we must borrow 42.86 at a risk-free rate.

9. The slope (Sharpe ratio) of the capital allocation line formed with the risky asset and the risk-free asset is \_\_\_\_\_\_\_\_\_\_.

A) 1.4

B) . 7

C) .5

D) .3

Answer: B. The slope of CAL is the Sharpe ratio of the risky asset. SR=(E(r)-rf)/r. Plugging in the numbers we obtain: SR=(0.12-0.05)/0.1=0.07/0.1=0.7.

10. A portfolio is composed of two stocks, A and B. Stock A has a standard deviation of return of 25% while stock B has a standard deviation of return of 5%. Stock A comprises 20% of the portfolio while stock B comprises 80% of the portfolio. If the variance of return on the portfolio is .0050, the correlation coefficient between the returns on A and B is \_\_\_\_\_\_\_\_\_\_.

A) -.225

B) -.474

C) .474

D) .225

Answer: D. We need to calculate a correlation coefficient between returns of two risky assets. This can be obtained from the formula for such a portfolio’s risk. P = w2A + (1-w)2B + 2w(1-w)ABcorr(A,B). In our question, P=0.0050, w=0.2, A=0.25, and B=0.05. Plugging in numbers, we have: 0.005=0.04\*0.0625+0.64\*0.0025+2\*0.2\*0.8\*0.25\*0.05\*corr(A,B). Solving, for corr(A,B), we get 0.225.

11. A portfolio is composed of two stocks, A and B. Stock A has a standard deviation of return of 5% while stock B has a standard deviation of return of 15%. The correlation coefficient between the returns on A and B is .5. Stock A comprises 40% of the portfolio while stock B comprises 60% of the portfolio. The variance of return on the portfolio is \_\_\_\_\_\_\_\_\_\_.

A) .0035

B) .0085

C) .0094

D) .0103

Answer: D. We can use exactly the same formula as in question 7. Hence, P=0.16\*0.0025+0.36\*0.0225+2\*0.4\*0.6\*0.05\*0.15\*0.5=0.0103.

12. Which of the following investments do you prefer? (A) Purchase a zero-coupon bond, which pays $1000 in ten years, for a price of $550. (B) Invest $550 for ten years in Chase at a guaranteed annual interest rate of 5.5%.

A) Investment A

B) Investment B

C) You are indifferent between the two

Answer: A. To determine the answer we need to compare the return one would get from each investment. B is given and equal to 5.5%. We need to find the return/yield of A. P=FV/(1+r)^T. Hence, r=(FV/P)^(1/T)-1. Plugging in the numbers, we obtain r=6.16% which is greater than 5.5%. Hence, one would prefer A over B.

Use the following to answer questions 13-15:

An investor can design a risky portfolio based on two stocks, A and B. Stock A has an expected return of 18% and a standard deviation of return of 20%. Stock B has an expected return of 12% and a standard deviation of return of 5%. The correlation coefficient between the returns of A and B is 50%. The risk-free rate of return is 10%.

13. The proportion of the optimal *risky* portfolio that should be invested in stock B is \_\_\_\_\_\_\_\_\_\_.

A) .2000

B) .2667

C) .3636

D) .8000

Answer: D. The most important insight to solve this question is that investors choose portfolios that maximize their Sharpe ratios. Sharpe ratio is defined as SR=E(r)/. We know that for two risky assets. E(rP) = w\*E(rA)+(1-w)\*E(rB). Likewise, P = w2A + (1-w)2B + 2w(1-w)ABcorr(A,B). This question, if asked as an open-ended question, would go beyond the class material as it would require taking a derivative of SR with respect to the portfolio weight w, and setting the derivative to zero (first-order condition). Your book gives you a solution to such a problem, but again I do not require that you memorize this solution, and you will not be asked to derive this in this class. However, since we are choosing from only 4 possible weights given in this question, we could simply plug in each of them into the SR formula and check which one gives us the highest Sharpe ratio. This process results in w=0.2 and 1-w=0.8.

14. The expected return on the optimal risky portfolio is \_\_\_\_\_\_\_\_\_\_.

A) 13.2%

B) 15.8%

C) 16.4%

D) 16.8%

Answer: A. Knowing w=0.2, we can easily calculate E(r)=0.2\*18%+0.8\*12% = 13.2%.

15. The standard deviation of return on the optimal risky portfolio is approximately \_\_\_\_\_\_\_\_\_\_.

A) 5%

B) 7%

C) 8%

D) 9%

Answer: B. Similarly, the risk of the portfolio will be P = w2A + (1-w)2B + +2w(1-w)ABcorr(A,B). Plugging in numbers, we obtain P = 7%.

**Part II: Detailed Questions**

**Question 2:**

*Microsoft is currently selling at $19.04 per share. You buy 1000 shares by using $15,000 of your own money and borrowing the remainder of the purchase price from your broker. The rate on the margin loan is 6%. The account has an initial margin of 50 percent and a maintenance margin of 25 percent.*

*(a) What is the percentage increase in the net worth of your brokerage account if the price of MSFT immediately changes to: (i) $25; (ii) $15?*

The purchase costs you $19,040. You borrow $4,040 from the broker and invest $15,000 of your own funds. The initial net value of the account is $15,000. Note that returns are always measured relative to the equity value you invest in your account.

* 1. The net account value increases to $25,000-$4,040=$20,960. Thus, the return is 20,960/15,000-1=39.73%
  2. The net account value falls to $15,000-$4,040=$10,960. Thus the return is 10,960/15,000-1= -26.93%

*(b) How low can the price of MSFT fall before you get a margin call?*

The value of the 1,000 shares is 1000\*P. Equity is 1000\*P – 4,040. You will receive a margin call when

.

*(c) How much money would you need to send your broker to satisfy the margin call if the stock price drops below the level computed in b?*

You will need to add cash until your margin equals the initial margin of 50 percent. The value of the securities is $5,390 (5.39\*1000) and you can borrow at most 50 percent of this (5390/2=2695). Thus, you need to pay back $1,345 (4040-2695) of the original margin loan of $4,040.

*(d) What is the rate of return on your margined position if MSFT is selling after one year at: (i) $25; (ii) $15? (You can ignore the dividend payments by MSFT).*

The margin loan with accumulated interest will be $4,040\*1.06=$4,282.40. The equity in your account is 1000P-$4,282.4. Therefore your rates of return are:

(i) 

(ii) 

**Question 3:**

Assume that it is now January 1, 2009.

(a) If you put $1,000 into a savings account at an 8 percent interest rate compounded annually on January 1, 2010 how much will you have in your account on January 1, 2013?

This is a question about Future Value. FV=1000\*1.08^3, or using your calculator 3, n, 8, i, 1000, PV, 0, PMT and then FV. FV= 1259.71.

(b) Suppose instead that you deposit the $1,000 in four payments of $250 each on the first of January in 2010, 2011, 2012, and 2013. How much will you have in your account on January 1, 2013?

FV=250\*1.08^3+250\*1.08^2+250\*1.08+250. Using your financial calculator, you put 4, n, 8, i, 0, PV, 250, PMT, and then FV. Note that you have to put n=4 because the payments are taken as of the end of the period. FV=1126.53

(c) Suppose that you make four payments of equal size on the first of January in 2010, 2011, 2012, and 2013. How large would each of those payments have to be in order to have the same ending balance you would have obtained by the single deposit of $1,000 on January 1, 2010?

First, the ending balance of investing 1000 is 1259.71 (see point a). So, we need to solve the following C\*1.08^3+C\*1.08^2+C\*1.08+C=1259.71, where C would be the payment we need to find. Analytically, C=1259.71/(1.08^3+1.08^2+1.08+1). Using your financial calculator: 4, n, 8, i, 0, PV, 1259.71, FV, and then we press PMT. C=279.56.

(d) Now assume that you put $1,000 into a savings account at an 8 percent interest rate compounded quarterly on January 1, 2010. How much will you have in your account on January 1, 2013?

We need FV=1000\*1.02^12. Note that we use the quarterly rate of 2% for 12 quarterly periods (3 years). Using your financial calculator: 12, n, 2, i, 1000, PV, 0, PMT, and then we press FV. FV=1268.24. Note that this is larger than the number calculated in (a), because of a higher frequency of compounding.

(e) If you need $1,000 by January 1, 2013 how much must you deposit on January 1, 2010? Only one deposit is to be made and the compounding rate is 8 percent annually.

This is the question for present value. PV=1000/1.08^3. Using your financial calculator: 3, n, 8, i, 1000, 0, PMT, 1000, FV, and then PV. PV=793.83.

(f) If you want to make equal payments on each January 1 from 2010 through 2013 to accumulate the $1,000 how large must each of the four payments be? The compounding rate is 8 percent annually.

We need to find C that makes the following hold. C\*1.08^3+C\*1.08^2+C\*1.08+C = 1000. Hence, C = 1000/(1.08^3+1.08^2+1.08+1). Using your financial calculator: 4, n, 8, i, 0, PV, 1000, FV, and then we press PMT. C=221.92.

(g) Suppose that you can deposit only $186.29 each January 1 from 2010 through 2013. What interest rate with annual compounding must you seek to achieve your goal of $1,000 by January 1, 2013?

We need to solve 186.29\*(1+r)^3+186.29\*(1+r)^2+186.29\*(1+r)+186.29=1000. Note that this equation would be hard to solve analytically. Using your financial calculator: 4, n, 0, PV, 186.29, PMT, -1000, FV, then i. r=20%.

**Question 4:**

*Assume that you manage a risky portfolio with an expected rate of return of 17% and a standard deviation of 27%. The T-bill rate is 7%. Suppose your client initially chooses to invest 70% of a portfolio in your fund and 30% in T-bill.*

1. *What is the expected return and standard deviation of your client's portfolio?*
2. *Now, suppose that your client wonders whether to switch the 70% that is invested in your fund to the passive portfolio with the expected return of 13% and a standard deviation of 25%. Explain to your client the disadvantage of the switch.*
3. *Show your client the maximum fee you could charge (as percent of the investment in your fund deducted at the end of the year) that would still leave him at least as well off investing in your fund as in the passive one. Hint: The fee will lower the slope of your client's CAL by reducing the expected return net of the fee.*

a) Expected return = .3\*7% + .7 \*17% = 14% per year.  
Standard deviation = .7 \* 27% = 18.9% per year.

b) With 70% of his money in my fund's portfolio the client gets a mean return of 14% per year and a standard deviation of 18.9% per year. If he shifts that money to the passive portfolio (which has a mean of 13% and standard deviation of 25%), his overall mean and standard deviation would become:

E(rc) = rf + .7(rM - rf)

In this case, rf = 7% and rM = 13%. Therefore,

E(rc) = 7% + .7 \* 6% = 11.2%

The standard deviation of the complete portfolio using the passive portfolio would be:

σc = .7 \* σM = .7 \* 25% = 17.5%

Therefore, the shift entails a decline in the mean from 14% to 11.2% and a decline in the standard deviation from 18.9% to 17.5%. Since both mean return *and* standard deviation fall, it is not yet clear whether the move is beneficial or harmful. The disadvantage of the shift is that if my client is willing to accept a mean on his total portfolio of 11.2%, he can achieve it with a lower standard deviation using my fund portfolio, rather than the passive portfolio. To achieve a target mean of 11.2%, we first write the mean of the complete portfolio as a function of the proportions invested in my fund portfolio, y:

E (rc) = 7 + y (17 - 7) = 7 + 10y

Because our target is: E(rc) = 11.2%, the proportion that must be invested in my fund is determined as follows:  
  
 11.2 = 7 + 10y ⇒ y = = .42  
The standard deviation of the portfolio would be:

σc = y \* 27 = .42 \* 27 = 11.34%.

Thus, by using my portfolio, the same 11.2% mean can be achieved with a standard deviation of only 11.34% as opposed to the standard deviation of 17.5% using the passive portfolio.

**Note!** An alternative and simpler way to see this is to compare Sharpe ratios of the two investments and choose the one with a higher value.

c) The fee would reduce the Sharpe ratio, i.e., the slope of the CAL. Clients will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee.

Slope of CAL with fee = =   
Slope of CML (which requires no fee) = = .24.

Setting these slopes equal we get:

= .24

10 - f = 27 \* .24 = 6.48  
  
 f = 10 - 6.48 = 3.52% per year

**Question 5:**

*Suppose that the economy is either in a recession next period or in a boom. Financial markets are frictionless (i.e., you can buy and sell assets without any constraints and transactions costs). Two assets, stocks and gold, are affected differently by the economic environment. The following table summarizes the rates of returns in these two states:*

|  |  |  |
| --- | --- | --- |
|  | *Boom* | *Recession* |
| *Return of Stocks* | *20%* | *-10%* |
| *Return of Gold* | *-10%* | *10%* |

1. *Show how to create a portfolio of stocks and gold that has zero risk.*

Let wGold be the proportion invested in gold and wStocks=1-wGold be the weight invested in stocks.

A portfolio of stocks and gold with zero risk should have the same return in booms as in recessions (thus no surprise in general):

RBoom=RRecession

wGold\*(-0.1)+(1-wGold)\*0.2 =wGold\*(0.1)+(1-wGold)\*(-0.1).

Solving for wGold gives wGold=60% and wStocks=1-wGold=40%. Let’s check that the return is identical in booms and recessions:

RBoom=wGold\*(-0.1)+(1-wGold)\*0.2=0.6\*(-0.1)+0.4\*0.2=0.02=2%

RRecession=wGold\*(0.1)+(1-wGold)\*(-0.1) =0.6\*(0.1)+0.4\*(-0.1)=0.02=2% Note that you did not have to assume anything about the probabilities of boom and recession to get it right.

1. *Suppose that risk-free T-Bills have a return of 4%. In light of results in a) is there any interesting investment opportunity here? If yes, please describe in detail the possible investment strategy and how it would make you a happy investor.*

The portfolio described in (a) has a return of 2% in the two possible future states. On the other hand, T-Bills have a return of 4%. We can make money by borrowing at 2% and investing the borrowed funds at 4%. This generates risk-free profits. To borrow funds at 2%, we short-sell the portfolio of 60% gold and 40% stocks. The difference of 2% is the risk-free profit we can pocket.